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Topologically nontrivial time-dependent chiral condensates *

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Abstract

Topologically nontrivial time-dependent solutions of the classical nonlinear σ model are studied as candidates of the disoriented chiral condensate (DCC) in 3+1 dimensions. Unlike the analytic solutions so far discussed, these solutions cannot be transformed into isospin-uniform ones by chiral rotations. If they are produced as DCCs, they can be detected by a distinct pattern in the angle-rapidity distribution of the neutral-to-charged pion ratio.

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1 Introduction

Analytic solutions of the classical nonlinear σ model [1, 2, 3] have been studied as candidates of the disoriented chiral condensates (DCCs) [4]. The solutions so far obtained are either configurations with spatially uniform isospin distribution or those which are chirally equivalent to them. When the isospin-uniform DCCs decay, the decay pions will obey the event-by-event pion charge distribution of $dP/df = 1/2\sqrt{f}$ in the neutral pion fraction f. In this paper we study as DCC candidates the time-dependent solutions of the nonlinear σ-model that are topologically nontrivial in the isospin-orbital space. Though the topologically nontrivial DCCs obey the same charge distribution dP/df as that of statistically random emission, the angle-rapidity distribution of pions should exhibit a very distinct experimental signature. We suggest a quantitative method of analysis to search for these DCCs.

2 Topologically nontrivial solutions

Let us express the pion field $\boldsymbol{\pi}(x)$ nonlinearly in terms of the scalar phase field $\theta(x)$ and the unit isovector field $\mathbf{n}(x)$ as

$$\boldsymbol{\pi}(x) = f_\pi \mathbf{n}(x)\theta(x), \tag{1}$$

where $f_\pi = 93$ MeV is the pion decay constant. Apart from the source term, the Lagrangian is given in terms of $\Sigma(x) = \exp(i\mathbf{n}(x)\cdot\boldsymbol{\tau}\,\theta(x))$ by

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma) + \frac{1}{2}\lambda f_\pi^2(\mathbf{n}^2 - 1), \tag{2}$$

in the chiral symmetry limit. After elimination of the Lagrange multiplier λ, the Euler-Lagrange equation reads [2]

$$\Box\theta - \sin\theta\,\cos\theta\,(\partial_\mu\mathbf{n}\cdot\partial^\mu\mathbf{n}) = 0, \tag{3}$$

$$\partial_\mu(\sin^2\theta\;\mathbf{n}\times\partial^\mu\mathbf{n}) = 0. \tag{4}$$

The analytic solutions so far known are either those with $\mathbf{n}(x) = \textit{constant}$ (the Anselm class) or those which are rotated to them by chiral transformations.

In this paper we explore the class of solutions whose $\mathbf{n}(x)$ fields point radially in the spacetime of $3+1$ dimensions:

$$\mathbf{n}(x) = \frac{\mathbf{r}}{r}\,. \tag{5}$$

We have obtained a hint for this postulate from the Skyrme assumption that led to the static soliton [5]. Unlike Skyrme, we do not need a stabilizing term in the Lagrangian since the static stability of solutions is irrelevant to us. The spherical symmetry of $\mathbf{n}(x)$ suggests we should choose $\theta(x)$ also to be spherically symmetric as

$$\theta(x) = \theta(t, r). \tag{6}$$

It is important to observe that with our postulate the Euler-Lagrange equation (4) for $\mathbf{n}(x)$ is automatically satisfied for any $\theta(t, r)$. The equation for $\theta(x)$ now reads

$$(\partial_t^2 - \nabla^2)\theta(t, r) + \frac{2}{r^2}\sin\theta(t, r)\,\cos\theta(t, r) = 0. \tag{7}$$

This wave equation allows many interesting solutions. All of them are topologically nontrivial since following the Skyrme model we can introduce the topological charge,

$$q = \int Q_0 d^3x, \tag{8}$$

where

$$Q_\mu(x) = \frac{\epsilon_{\mu\nu\kappa\lambda}}{24\pi^2}\mathrm{tr}(X^\nu X^\kappa X^\lambda), \tag{9}$$

with $X^\nu = \Sigma^\dagger\partial^\nu\Sigma$. The current Q_μ is locally conserved, $\partial^\mu Q_\mu = 0$, and q is invariant under chiral $SU(2)\times SU(2)$ rotations. The charge q is nonvanishing for our solutions while it is zero for the isospin uniform solutions. It should be noted that q is time dependent when we compute it for the pion fields alone since the current Q_μ flows from the shell of hadron debris into the DCC. Actually it is not even finite since our pion fields are singular as we approach the light cone. This should not bother us since the nonlinear σ model after all does not apply to the close neighborhood of the light cone where kinetic energy is too large. Our purpose of mentioning the topological charge q here is that our solutions are chirally inequivalent to the isospin-uniform solutions. When we adopt our solutions as DCC candidates, we do not accept the Skyrme model of baryons [6] in which the charge q is identified with the baryon number. If we did, our DCCs would be loaded with nucleons or antinucleons.

Let us solve for $\theta(t,r)$ by restricting the form of $\theta(t,r)$. Since θ is dimensionless and its equation of motion is scale invariant, a simple case of interest is that $\theta(t,r)$ is a function only of the ratio of r and t:

$$\theta(t,r) = \theta(\xi), \qquad (\xi \equiv \frac{r}{t}). \tag{10}$$

The wave equation then turns into

$$\frac{d^2}{d\xi^2}\theta + \frac{2}{\xi}\theta - \frac{2}{\xi^2(1-\xi^2)} \sin\theta\, \cos\theta = 0. \tag{11}$$

It can be easily solved numerically. The behavior at $\xi = 0$ is determined by the singularity at $\xi = 0$ in the wave equation (11). Barring a singularity at $r = 0$ for $\theta(x)$ since a source does not exists at $r = 0$ after $t = 0$, we determine that $\theta(\xi) \propto \xi$ as $\xi \to 0$. By giving one more boundary condition, $d\theta/d\xi$ at $\xi = 0$, we can compute a profile of the scalar phase field $\theta(\xi)$. In Fig.1 we have plotted $\theta(\xi)$ for a few different values of $\theta'(0)$. On the light cone ($\xi = 1$), $\theta(t,r)$ is singular because of $1/(1-\xi^2)$ in the third term of Eq.(11), but only in the immediate neighborhood of $\xi = 1$. The function $\theta(\xi)$ is smooth and monotonic practically everywhere inside the light cone.

When $\theta(t,r)$ is not a function only of $\xi = r/t$, analyticity at $r = 0$ requires the behavior $\theta(t,r) \to r$ at $r \to 0$, not necessarily $\theta(t,r) \to r/t$. We have drawn the asymptotic configuration $\theta(\infty, r)$ in Fig.2. The static equation for $\theta(\infty, r)$ is invariant under rescaling of r. As we know from the Skyrme model, this $\theta(\infty, r)$ is not a local minimum of energy with respect to rescaling of r since there is no Skyrme term in our Lagrangian. In the dynamical case under considerations, solutions of all different scales are acceptable for $\theta(\infty, r)$. The amount of energy fed in by the hadron debris determines the scale of a DCC that is produced.

From the viewpoint of total energy, the nontrivial $\mathbf{n}(\mathrm{x})$ costs nothing since $\Sigma(x)$ remains at the bottom of the double-well potential and the static $\mathbf{n}(\mathbf{r})$ does not contribute to kinetic energy. As far as energy is concerned, the topologically nontrivial solutions are no different from the isospin-uniform solutions.

3 Isospin property

It might appear that the topologically nontrivial DCCs have zero isospin because of the *spherical symmetry*. It is not correct. At the quantum level they are not eigenstates of $I = 0$ but superpositions of eigenstates of many different isospin values. To see it, we should represent the quantum state of the classical configuration by the coherent state [7, 8]:

$$|\mathbf{n}(\mathbf{r})\theta(t,r)\rangle = \exp\Big(-if_\pi \int \mathrm{tr}\Big(\theta(t,r)\mathbf{n}(\mathbf{r})\cdot\partial_t\boldsymbol{\pi}(x) - \partial_t\theta(t,r)\,\mathbf{n}(\mathbf{r})\cdot\boldsymbol{\pi}(x)\Big)d^3x\Big)|0\rangle, \tag{12}$$

where $\boldsymbol{\pi}(x)$ is the isovector quantum pion field. The exponent is invariant under the simultaneous isospin-orbital rotations generated by $\mathbf{K} = \mathbf{I} + \mathbf{L}$, where $\mathbf{L}$ is orbital angular momentum. Therefore this state is an eigenstate of $K = 0$, not of $I = 0$ nor $I_3 = 0$ [9]. The exponent of Eq.(12) can be expressed in terms of the creation operators $a^{(\alpha)\dagger}_{klm}$ of pions with charge $\alpha = (+, -, 0)$, energy $k(\equiv |\mathbf{k}|)$, and orbital angular momentum (l, m) as

$$\propto \int \Big(a^{(-)\dagger}_{k11} + a^{(0)\dagger}_{k10} + a^{(+)\dagger}_{k1\,-1}\Big)\sqrt{2k}\,\tilde{\theta}_l(k)kdk \;-\; h.c., \tag{13}$$

where $\tilde{\theta}_l(k)$ is the Bessel transform $\int \sqrt{kr}\,J_{l+1/2}(kr)\theta(0,r)r\,dr$. Projection onto the N_π pion state of Eq.(12) is N_π-th power of Eq.(13) operated on $|0\rangle$. By isospin decomposition we see that the state is not purely an isosinglet but has a wide distribution in I. It is remarked that the initial state of $p\bar{p}$ collision has $K = 0$ or 1 in the case of head-on collision with zero impact parameter since $L = 0$ and $I = 0$ or 1.

4 Momentum distribution of neutral-to-charged pion ratio

We study the spectrum of the pions decaying from the topologically nontrivial DCCs. We focus on the correlation between the isospin and momentum distributions since it shows a distinct characteristic. Let us describe the expanding hadron debris of the baked Alaska scenario [10] by the isovector source $\boldsymbol{\rho}(x)$ $(= \Box\boldsymbol{\pi}(x))$. The standard method [11] gives the momentum spectrum of the

pions with the Cartesian isospin component i and momentum $\mathbf{k}$ as

$$2k_0 \frac{dN_i}{d^3\mathbf{k}} = \frac{1}{(2\pi)^3} |\tilde{\boldsymbol{\rho}}(\mathbf{k}) \cdot \mathbf{e}_i|^2, \tag{14}$$

where $\tilde{\boldsymbol{\rho}}(\mathbf{k})$ is the four-dimensional Fourier transform with $k_0 = |\mathbf{k}|$ for massless pions,

$$\tilde{\boldsymbol{\rho}}(\mathbf{k}) = \int \boldsymbol{\rho}(x)\, e^{ikx} d^4x, \tag{15}$$

and $\mathbf{e}_i$ is a unit Cartesian isospin vector along the i-th direction. The spherically symmetric pion fields, $\theta(t,r)$ and $\mathbf{n}(x) = \mathbf{r}/r$, can be produced only by $\boldsymbol{\rho}(x)$ of the same symmetry. Therefore $\boldsymbol{\rho}(x)$ may be expressed as

$$\boldsymbol{\rho}(x) = \frac{\mathbf{r}}{r} \rho(r) \delta(t^2 - r^2). \tag{16}$$

Then the $\mathbf{k}$ dependence of its Fourier transform is determined in the form

$$\tilde{\boldsymbol{\rho}}(\mathbf{k}) = \frac{\mathbf{k}}{|\mathbf{k}|} \tilde{\rho}(|\mathbf{k}|). \tag{17}$$

Substituting Eq.(17) in Eq.(14), we find that the pion isospin is correlated with the momentum direction as

$$2k_0 \frac{dN_i}{d^3\mathbf{k}} \propto (\mathbf{k} \cdot \mathbf{e}_i)^2. \tag{18}$$

Namely, neutral pions are emitted preferentially along the z-axis while charged pions are into the xy-plane. Note however that the z-axis does not necessarily coincide with the collision axis. For some DCC, the z-axis happens to be the collision axis. Then many other DCCs can exist that are related to this one by isospin rotations. They form one isospin family of DCC solutions. When the z-axis coincides with the collision axis, we expect to see a pair of parallel π^0-rich bands around y_1 and y_2 in the ϕ-y plot, where ϕ is the azimuthal angle of the pion momentum $\mathbf{k}$ around the collision axis and y is the rapidity variable. The region between the two bands is filled dominantly with charged pions. When the isospin axis is not parallel to the collision axis, a pair of π^0-rich domains is found at (ϕ, y_1) and ($\phi + \pi$, y_2) in the ϕ-y plot. The two π^0-rich domains are separated by a $\pi^{\pm}$-rich region (Fig.3).

The event-by-event distribution dP/df of the neutral pion fraction $f = N_{\pi^0}/(N_{\pi^+} + N_{\pi^-} + N_{\pi^0})$ can be obtained from Eq.(18). By integrating over the

angles of $\mathbf{k}$, we find that the particle multiplicity is equal for all three pion charge states. Therefore the distribution dP/df cannot distinguish the topologically nontrivial DCC events from random emission events.

We can make search of the topologically nontrivial DCCs more quantitative. We should first select the DCC candidates, for instance, by abundance of soft p_t pions and find their approximate overall rest frames. To enhance the signature, we should select only those events with $f \approx 1/3$. We then determine event by event the isospin *polar* direction $\hat{\mathbf{z}}$, namely the π^0-dominant direction in the momentum space, by maximizing a suitably defined quantity as follows: By choosing a tentative $\hat{z}$ direction, compute for π^0 and $\pi^\pm$

$$\begin{aligned} C_0 &= \sum_{i=\pi^0} (\hat{\mathbf{k}}_i \cdot \hat{\mathbf{z}})^2, \\ C_\pm &= \sum_{i=\pi^\pm} (\hat{\mathbf{k}_i} \times \hat{\mathbf{z}})^2, \end{aligned} \tag{19}$$

where $\hat{\mathbf{k}}_0$ and $\hat{\mathbf{k}}_\pm$ are the unit vectors along neutral and charged pion momenta, respectively, and the summations are taken over all DCC pions from each event ($N_{\pi^0} \approx N_{\pi^+} \approx N_{\pi^-}$). Then vary the direction of $\hat{z}$ so as to maximize the product

$$C = C_0\, C_\pm. \tag{20}$$

The quantity C takes the maximum value when $\hat{z}$ coincides with the isospin polar axis. Let this maximum be C_{max}. Then compute C for the same $\hat{z}$ direction by interchanging $\pi^\pm$ and π^0. Let us call it C_{min}. Then the ratio

$$S = \frac{C_{max} - C_{min}}{C_{max} + C_{min}} \tag{21}$$

is equal to 5/7 for the topologically nontrivial DCCs with sufficiently large N_π. As the direction of $\hat{z}$ is varied, S sweeps between 5/7 and $-5/7$. In contrast, S is independent of the direction of $\hat{z}$ and equal to zero for the isospin-uniform DCCs as well as for random emission. Feasibility of this test depends on how large N_π is. Since the statistical errors are of $O(1/\sqrt{N_\pi})$, $N_\pi \gtrsim 25$ will do.

5 Chiral rotations and disorientation

The topologically nontrivial solutions of the form $\theta(\xi)$ behave like $\sim \xi$ at $\xi(= r/t) \to 0$. At sufficiently large t, therefore, the phase pion field $\theta(\xi)$ approaches zero, that is, the background vacuum relaxes to the true vacuum at any fixed location off the light cone. If the DCC is defined to be the disoriented condensate that would approach a *wrong* vacuum at $t \to \infty$, one might not want to call such a condensate as the DCC [3]. When we make chiral rotations on $\theta(\xi)$, we obtain infinitely many more solutions that carry the same topological charge but different vector-isospin distributions. By actually performing the axial-isospin rotations, we can obtain solutions whose $\mathbf{n}(x)$ is not spherical at finite t. As $t \to \infty$, these rotated solutions approach static limits with uniform isospin orientations at all finite locations:

$$\lim_{t\to\infty} \mathbf{n}(x)\theta(x) = \mathbf{n}_0\theta_0, \tag{22}$$

where $\mathbf{n}_0$ is the axis of an axial-vector isospin rotation ($\mathbf{n}_L = \mathbf{n}_R \equiv \mathbf{n}_0$) and θ_0 is its rotation angle ($\theta_L = -\theta_R \equiv \theta_0$) [2, 3]. Namely, the asymptotic disorientation stays nonvanishing and turns uniform.

When $\theta(t, r)$ is not a function only of $\xi = r/t$, we have found $\theta(t, r) \to r$ at $r \to 0$, instead of $\to r/t$. It remains *disoriented* and *nontrivial* as $\mathbf{n}(x) = \mathbf{r}/r$ even at $t = \infty$. Chiral rotations transform $\mathbf{n}(x) = \mathbf{r}/r$ into nonspherical $\mathbf{n}(x)$.

6 Explicit chiral symmetry braking

In the presence of an explicit chiral symmetry breaking, a DCC cannot really reach its asymptotic limit predicted by the chiral symmetric wave equations. It starts decaying before its kinetic energy $f_\pi^2\dot{\theta}^2/2$ becomes comparable with the potential difference of the symmetry breaking $m_\pi^2 f_\pi^2(1-\cos\theta)$. However, the postulate of $\mathbf{n}(x) = \mathbf{r}/r$ is still compatible with the explicit chiral symmetry breaking. Furthermore the topological charge q is defined in the same form and the local current conservation $\partial^\mu Q_\mu = 0$ remains valid. The only change is appearance of a scale breaking term proportional to m_π^2 in the wave equation of

$\theta(t,r)$:

$$\Box\theta + \frac{2}{r^2}\sin\theta\,\cos\theta = m_\pi^2 \sin\theta. \tag{23}$$

With the explicitly scale dependent term present, we can no longer postulate $\theta = \theta(\xi)$. Instead $\theta(x)$ is a function of two dimensionless variables r/m_π and t/m_π. But the wave equation still allows a spherically symmetric $\theta(t,r)$, and the behavior $\theta(t,r) \sim r$ at $r \to 0$ is not affected by the symmetry breaking. As long as the form of $\mathbf{n}(\mathbf{r})$ is the same, our prediction in the ϕ-y distribution of the decaying pions is valid with no modification.

7 Conclusion

We have argued that topologically nontrivial DCCs are not only possible but also have an equally good or bad change of being produced as the isospin-uniform ones. If they are actually produced, they will show a clear experimental signature in the ϕ-y plot of the decay pions. In order to produce them the hadron debris must also expand with a spherically symmetric isospin distribution $\mathbf{J}_0 \propto \hat{\mathbf{r}}$ according to geometrical symmetry. No dynamical mechanism is known that suppresses such a configuration of hadron debris. In a DCC search the topologically nontrivial DCCs should also be searched for by the analysis proposed here.

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Figure captions

Fig.1: The scalar phase function $\theta(\xi)$ for a few different boundary values of $\theta'(0)$. $\theta(\xi) \propto \xi$ is required at $\xi \to 0$ by the wave equation.

Fig.2: The asymptotic configuration of $\theta(t, r)$ at $t = \infty$ in the case that $\theta(t, r)$ is not a function only of $\xi = r/t$. The variable r is expressed in the unit of m_π.

Fig.3: Schematic pictures of the pion charge distribution in the ϕ-y plot for a topologically nontrivial DCC whose I_3 direction is along the collision axis (3a) and off the collision axis (3b).